Models vs. Reality – Problems, Modelling, Challenges

Jürgen Kurths

Institut für Physik, Humboldt-Universität, Berlin

&

Potsdam-Institut für Klimafolgenforschung

POTSDAM INSTITUTE FOR CLIMATE IMPACT RESEARCH

PIK

juergen.kurths@pik-potsdam.de

Main Collaborators:

B. Bookhagen, S. Breitenbach, J. Donges, R. Donner, B. Goswami, J. Heitzig, N. Malik, N. Marwan, P. Menck, K. Rehfeld, C. Zhou, Y. Zou

Outline

- Historical problems and brilliant modelling approaches
- Recent additional problems
- Beginning of systematic modelling – linear systems science
- More advanced modelling – complex systems science: from nonlinear dynamics to complex networks
- Challenges (open problems) and Outlook

What is the Earth´s Human Carrying Capacity?

Condition: Appropriate for both – Earth and humans

The epoch of brilliant approaches

First **conceptual models**

Modelling Problem

- **first principles are known in physics** or chemistry - basic laws (well accepted), e.g. mechanics, electromagnetism, fluid dynamics, atmospheric dynamics
- But **NOT in socio-economy** etc.

➔ Modelling needs different approaches, in particular: **conceptual** models vs.
formal (mathematical) models

Forecasting maximum world population possible

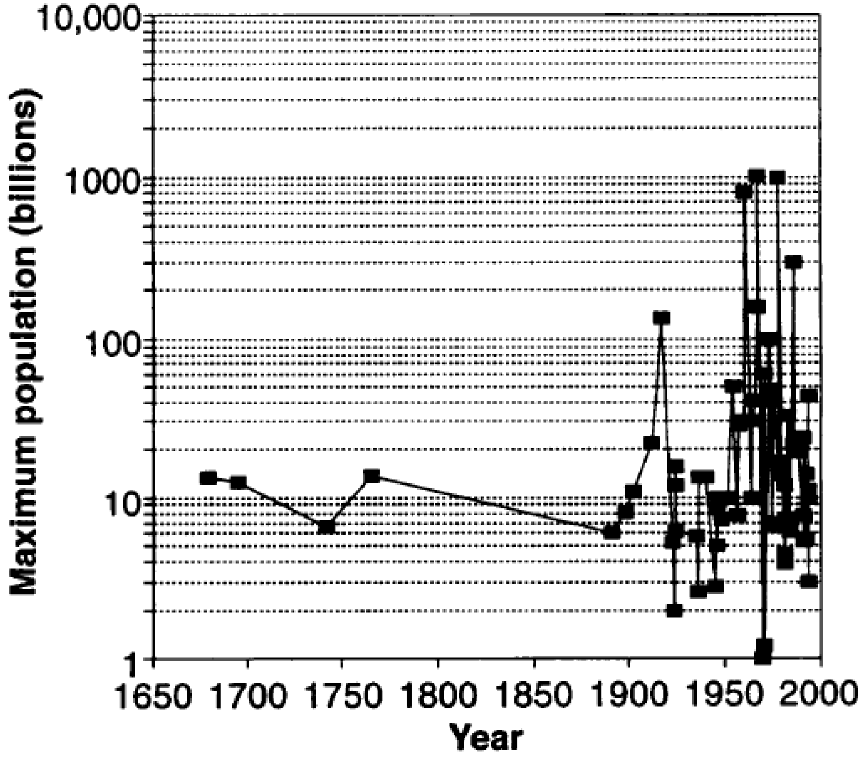


Fig. 3. Estimates of how many people Earth can support, by the date at which the estimate was made. When an author gave a range of estimates or indicated only an upper bound, the highest number stated is plotted here (*55*).

Recent estimates vary between

< 1 Billion and

> 1000 Billion

J. E. Cohen, Science (1995)

First Estimate: Antoni van Leeuwenhoek (1679)

His Approach (model): $N = N(h) * R$

- population of Holland N(h) that time (1 Million people)
- R ratio of Earth´s inhabited land area to Holland´s area (he estimated as 13,385)
- Result: 13.4 Billion
- Note that this is

  - a wrong model

  - with wrong specific parameters,

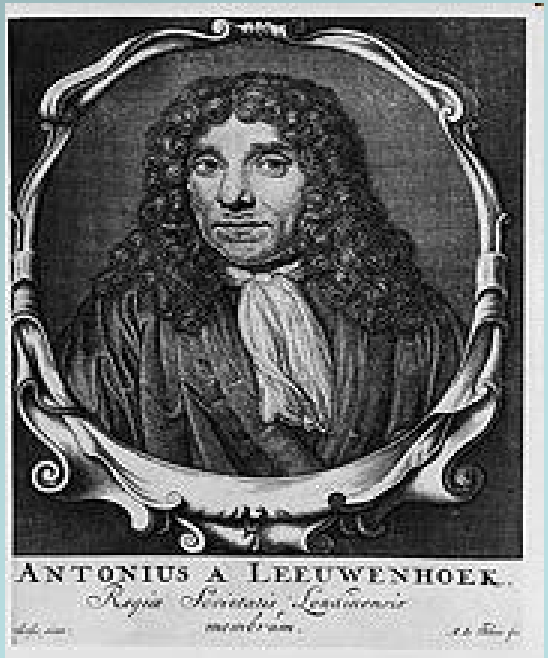
but leading to an acceptable result (from today´s knowledge)!!!

Today´s Parameters

- N(h) = 16.493.156 (Jan 2009)
- R = 148.900.000 / 41.528 = 3.585,5

➔ N = 59.136.749.383 (59 billion)

A. Leeuwenhoek – a serious scientist



Eichmeister (calibrator) and Landvermesser (surveyor) in Delft

built microscopes with high precision

Several discoveries in biology, e.g.

-bacteria in his mouth,

-fleas and mussels are from eggs (not spontaneously from sand or dirt)

Conclusion 1

Infering Models for Diagnostics/Predictions even from Data – A substantial puzzle

(and understanding underlying phenomena)

is highly non-trivial. It requires more than data mining techniques

Next step: include time evolution

Population Dynamics

Similar approach, but a bit more advanced:

N(t) population at time t

Rate of change: dN/dt = births – deaths + migration

A) Simple case: no migration, birth and death proportional to N:

dN/dt = bN – dN ➔ N(t) = N(0) exp (b-d)t

b, d > 0, N(0) initial population

if **b > d – population gows exponentially**
b > d “ dies out

Too simple??? World population in billions

mid17th	19th	1927	1960	1974	1987	2000	2050	2100
0.5	1	2	3	4	5	6.3	10*	11.2*

Exponential since 1900, but not forever (probably…)!!!

Conclusion 2:

Some restrictions necessary to include

Adjustment to exponential growth –

self-limited process (Verhulst 1838)

Logistic Growth

$$dN/dt = r N (1 - N/K)$$

per capita (pro kopf) birth rate: 1-N/K

K carrying capacity of environment

∞

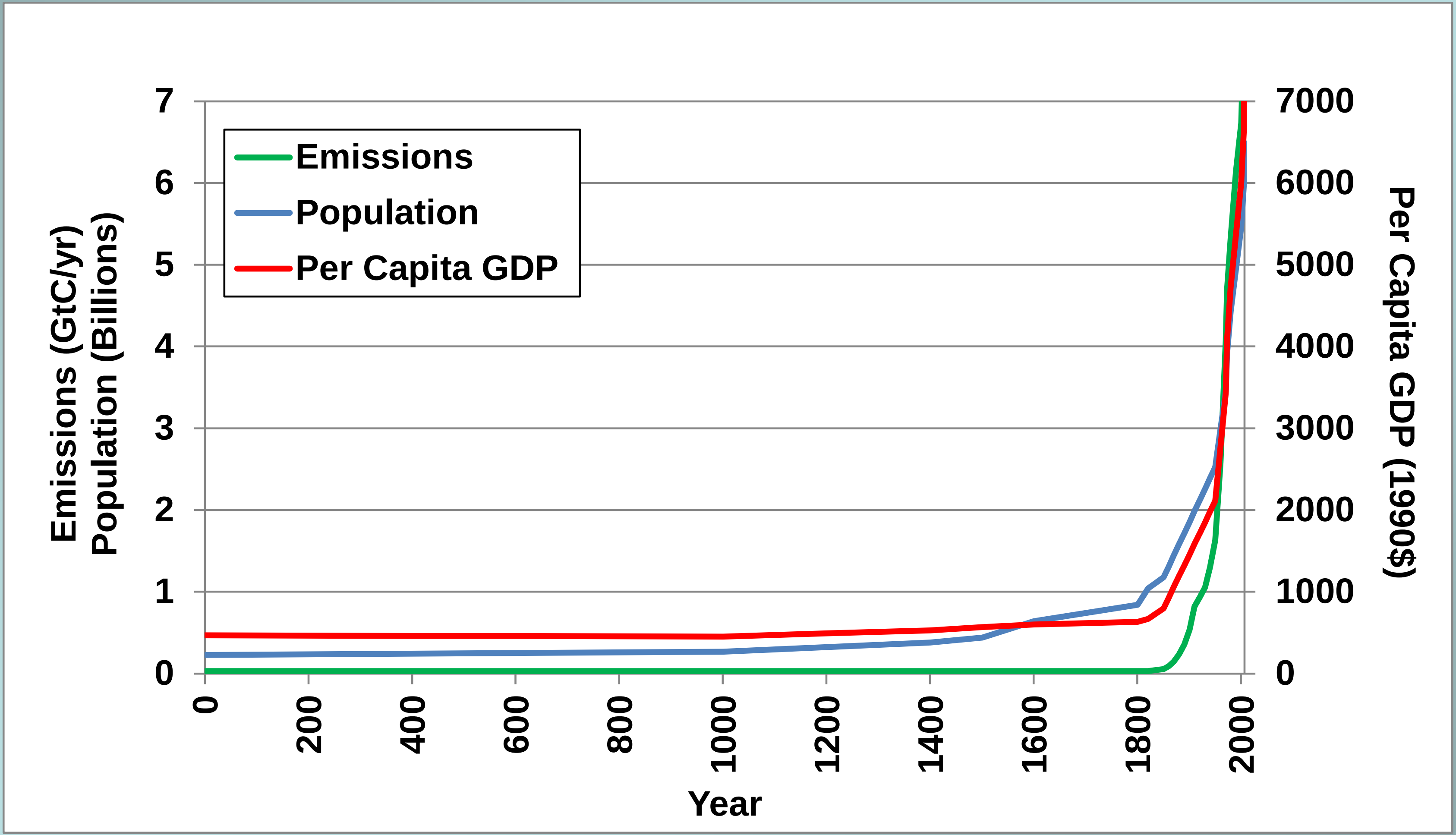
Two **steady** states N = 0 (unstable) ,
N = K (stable)

$N(t) = N(0) K \exp(rt) / \{ K + N(0) [\exp(rt) - 1]\} \rightarrow K$ as $t \rightarrow \infty$

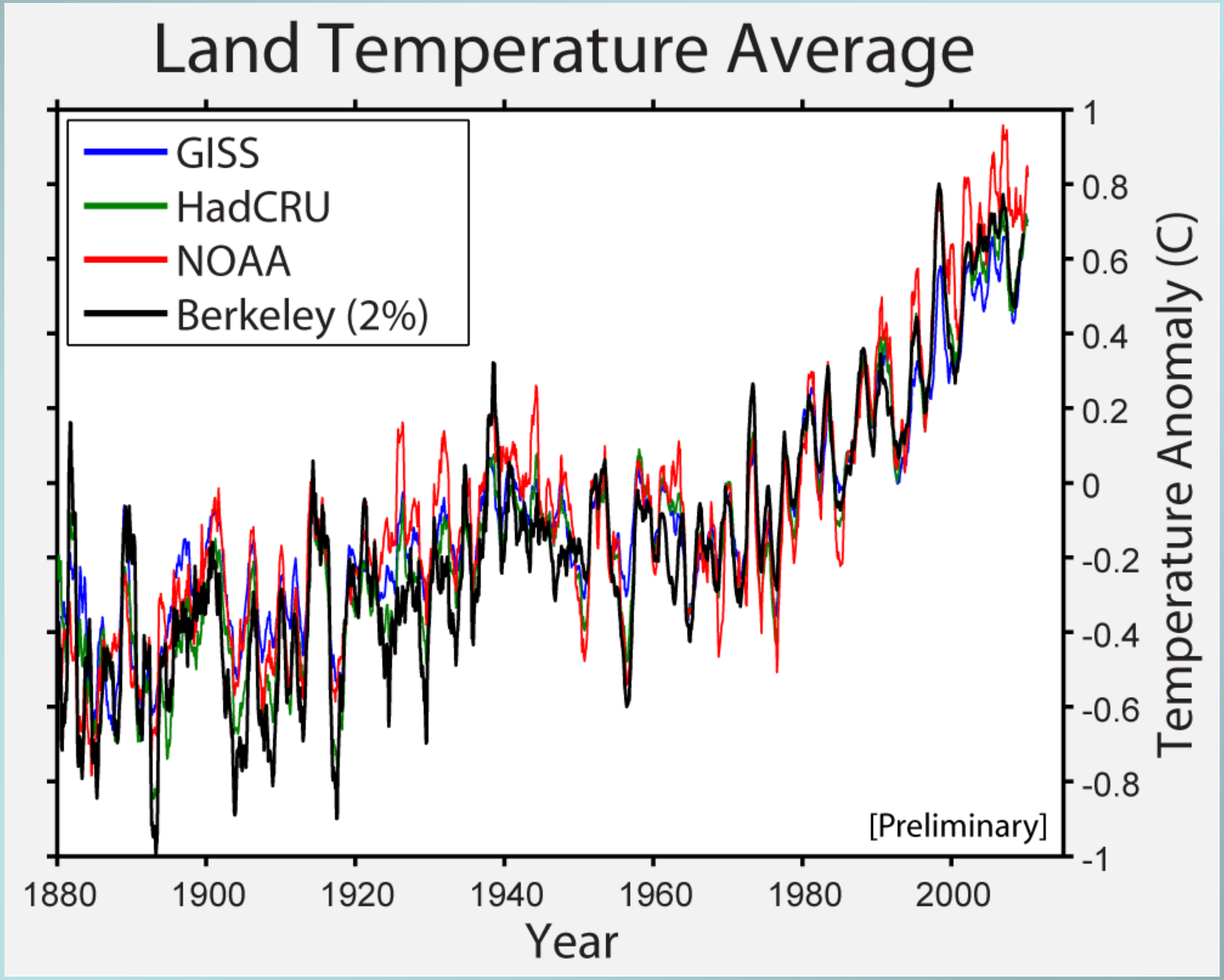
Recent: additional factors to include

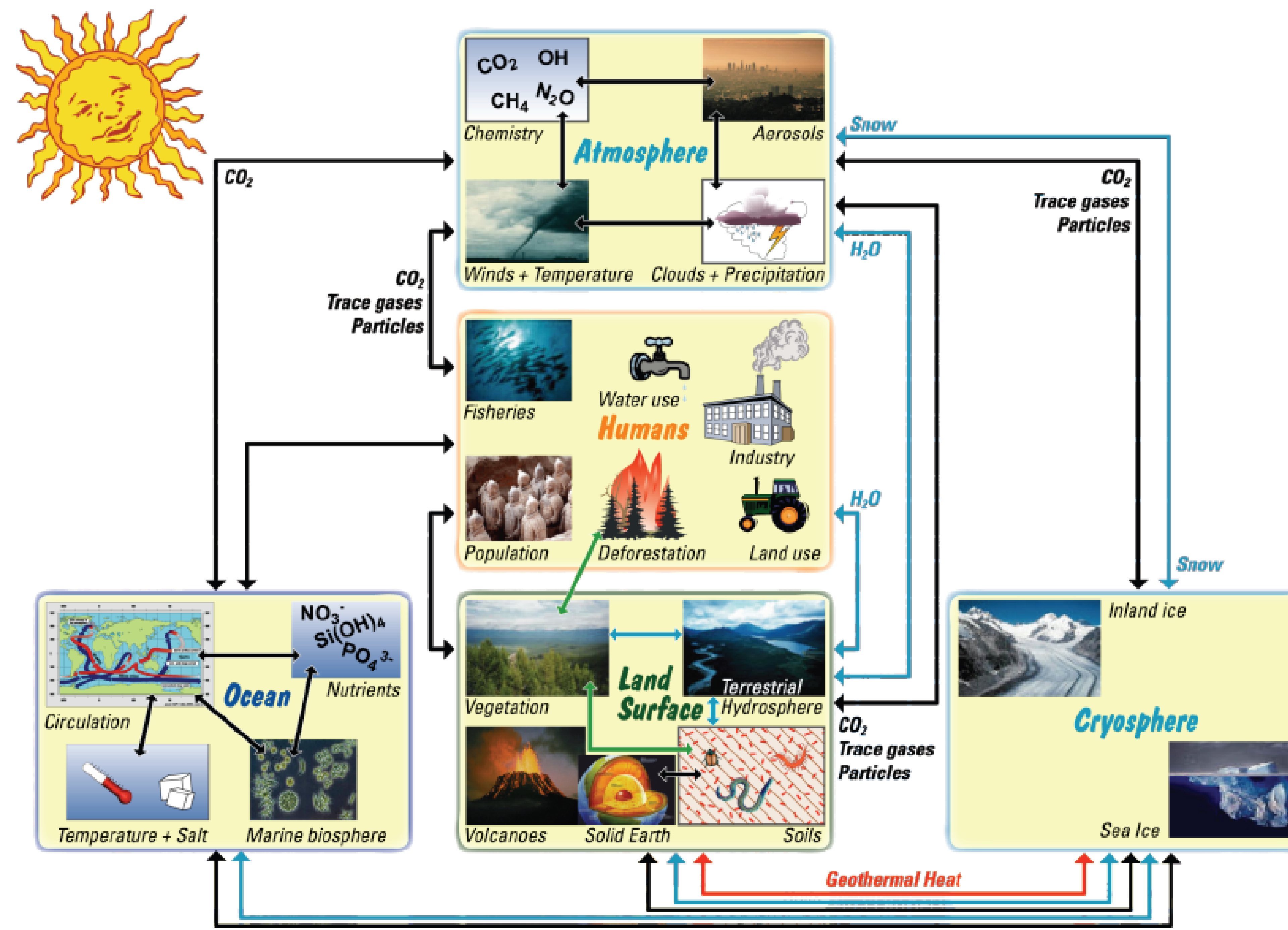
- Further resources: **Energy**
- **Present to humans** around 1800: fossile energy (coal, gas, oil)

Population Growth vs. Emission



Long term trends show clear evidence of increase





CO_2 OH
CH_4 N_2O
Chemistry
Aerosols
Atmosphere
Winds + Temperature
Clouds + Precipitation
Snow
CO_2
CO_2
Trace gases
Particles
H_2O
CO_2
Trace gases
Particles
Fisheries
Water use
Humans
Industry
Population
Deforestation
Land use
H_2O
Snow
NO_3^-
$Si(OH)_4$
PO_4^{3-}
Nutrients
Ocean
Circulation
Temperature + Salt
Marine biosphere
Vegetation
Land Surface
Terrestrial Hydrosphere
Volcanoes
Solid Earth
Soils
CO_2
Trace gases
Particles
Inland ice
Cryosphere
Sea Ice
Geothermal Heat

Challenge:

Built: Earth System Model

(Whole Earth Model,
Integrated Assessment Model)

To include various interactions
(feedbacks)

Use of (super) computers

Computers are useless. They only provide answers.

(Pablo Picasso)

Conclusion 3: to discuss in working group

The epoch of systematic mathematical approaches

(Linear) Systems Science – First **Formal Models** (1960ies -80ies)

Black-Box-Models

$$y = A\,x$$

Model (Operator) A

- Regression (most linear)
- Differential equation (most linear)

Typical mathematical problem:

- Given: output y
- Wanted (to estimate): input x and model (parameter) A
- Inverse problem (mostly ill-posed) – regularization techniques

Stochastic Model: y = A x + noise

- Type 1: **autoregressive processes** (order p)

$$X_t = f_1 X_{t-1} + \ldots + f_p X_{t-p} + Z_t \quad \{Z_t\} \sim WN(0,\sigma^2)$$

- Type 2: **moving average processes** (order q)

$$X_t = Z_t + q_1 Z_{t-1} + \ldots + q_q Z_{t-q}, \quad \{Z_t\} \sim WN(0,\sigma^2).$$

- Type 3: **ARMA processes (order p, q)**

$$X_t - f_1 X_{t-1} - ... - f_p X_{t-p} = Z_t + q_1 Z_{t-1} + ... + q_q Z_{t-q}$$

- Type 4, 5….
- Mostly: linear, causal (invertible)

Autoregressive models

- Simple recursive parameter estimation

$$\alpha_{M+1,M+1} = \frac{\rho_{M+1} - \sum_{s=1}^{M} \alpha_{s,M} \rho_{M+1-s}}{1 - \sum_{s=1}^{M} \alpha_{M+1-s,M} \rho_{M+1-s}}$$

- Well developed order selection (p, q) techniques for ARMA

Order Selection/Model Identification

Modell-Selektionskriterien für ARMA$[p, q]$-Prozesse mit Residualvarianz $\hat{\sigma}^2_{p,q}$ sind

(1) **das AIC-Kriterium** (Akaike's Information Criterion)

$$\mathrm{AIC}(p, q) := \ln \hat{\sigma}^2_{p,q} + 2\,\frac{(p+q)}{N}\,, \qquad [6.3.3.5]$$

(2) **das BIC-Kriterium** (Bayesian Information Criterion)

$$\mathrm{BIC}(p, q) := \ln \hat{\sigma}^2_{p,q} + \frac{(p+q)\ln N}{N}\,, \qquad [6.3.3.6]$$

(3) **das HQ-Kriterium** (Hannan-Quinn-Kriterium)

$$\mathrm{HQ}(p, q) := \ln \hat{\sigma}^2_{p,q} + \frac{2(p+q)\cdot c\cdot \ln(\ln N)}{N} \text{ mit } c > 1. \qquad [6.3.3.7]$$

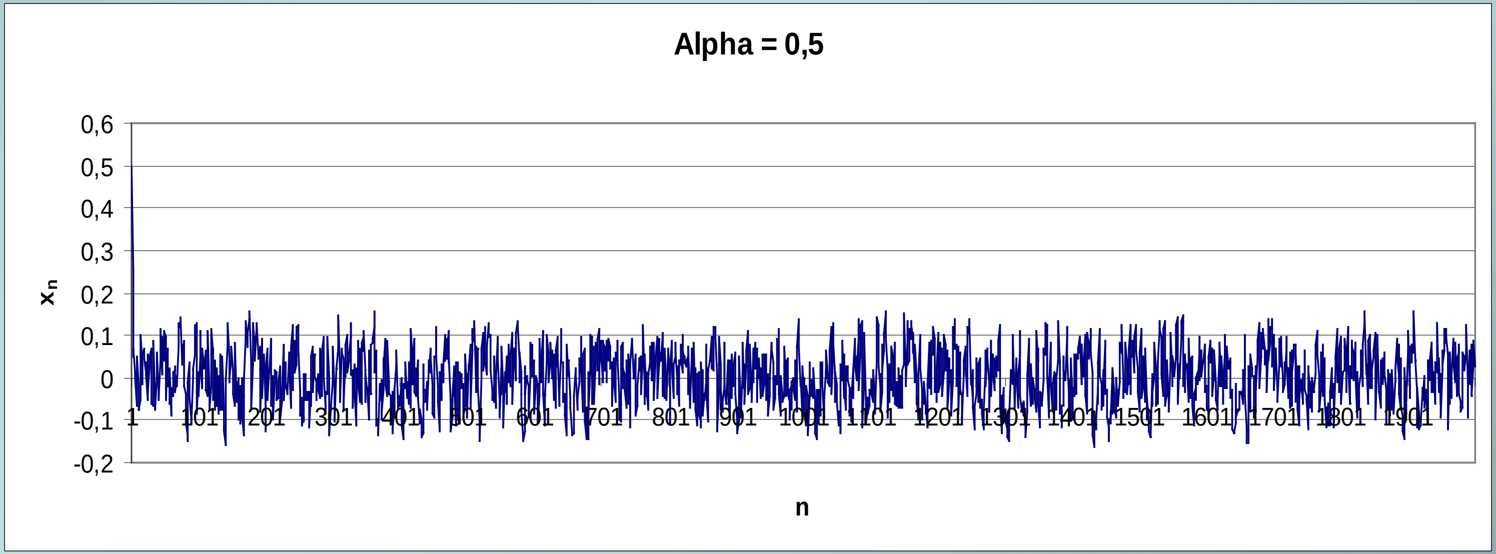
Auszuwählen ist dabei jeweils dasjenige ARMA$[p, q]$-Modell, für welches das verwendete Kriterium minimal ist.

General Properties

- Well developed statistical evaluation (tests of significance)
- Instructive presentation in frequency domain (power spectra)
- Applicable for rather short observations (time series) ➔ sliding (windowed) analysis of „changing“ processes (non-stationary)
- Generalized to multivariate processes (several parameters)

Autoregressive p = 1

$$X_t = \alpha X_{t-1} + Z_t \ , \ X_0 = 0.5, \alpha = 0.5, \sigma = 0.1$$



Problems

- No nonlinear feedbacks between different subsystems possible (typical situation in most applications)
- Nonlinear self-limited growth not included (e.g. logistic growth)
- Generated dynamics rather simple – not complex

➔Conclusion 4: linear black box approach very limited potential for our purpose

Complex Sytems Science –

Part 1: Nonlinear Dynamics (1980ies – about 2000)

Low-dimensional nonlinear systems (feedbacks)

Paradigmatic example: Logistic Map

$$x_{n+1} = r\, x_n\, (1 - x_n)$$ nonlinear difference equation

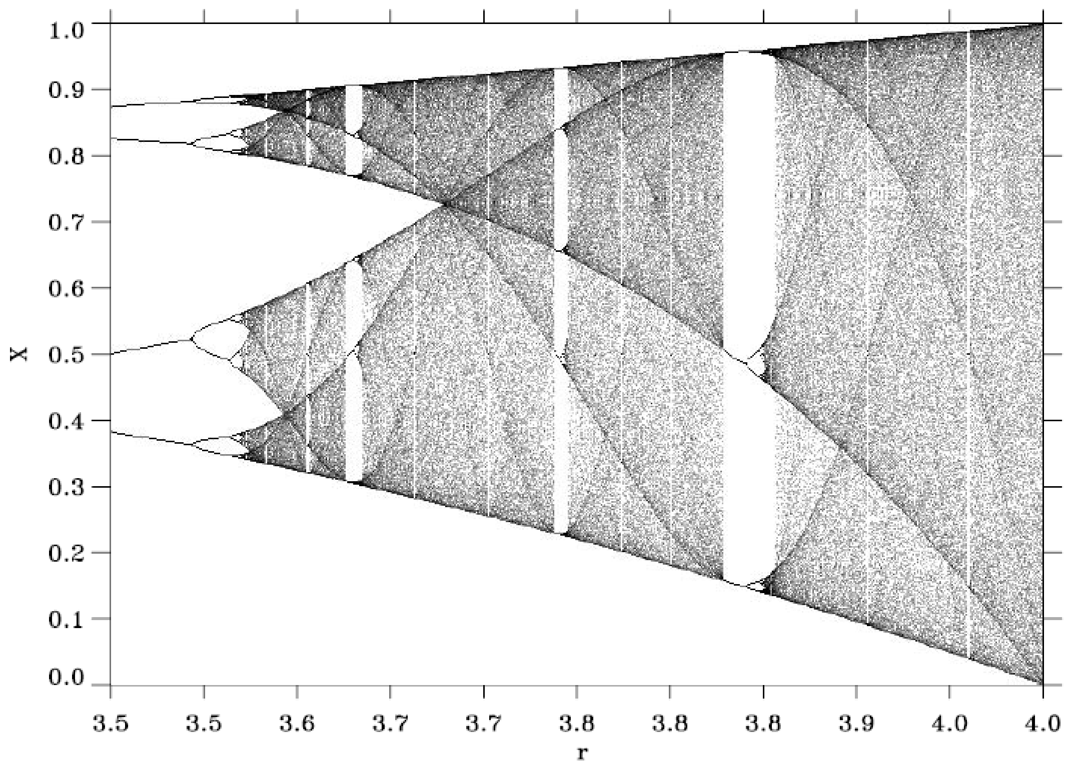


Figure 1: Bifurcation diagram for the logistic map in the interval [3.5,4] (Feigenbaum diagram).

New Methods & Phenomena

- Fractal objects (fractal dimensions)
- Deterministic Chaos
- Limited Predictability (Lyapunov exponents)
- Rich Dynamics (steady state, periodic, quasi-periodic, chaotic, intermittent)
- Rapid Qualitative Transitions – Bifurcations – Tipping Points (regular – chaos, chaos – chaos)

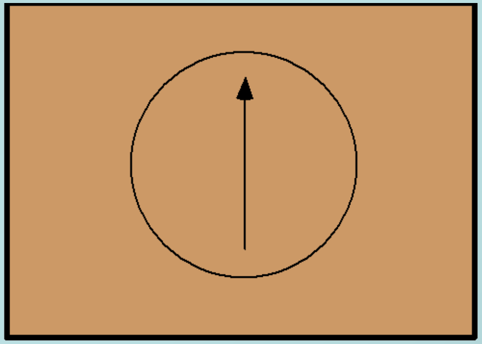
New Methods & Phenomena

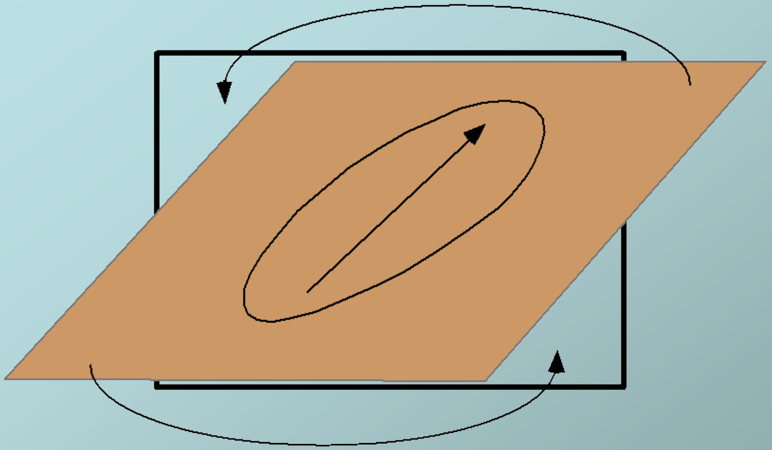
- Noise-induced Order (stochastic resonance - SR, coherence resonance - CR)
- Complex Synchronization (complete, generalized, phase)
- Recurrence (but not long-term predictable)

Poincaré's Recurrence

Crutchfield 1986,
Scientific American

Arnold's cat map





Poincare‘s Recurrence - demo

Bridge Opening

- Unstable modes always there
- Mostly only in vertical direction considered
- Here: extremely strong unstable lateral Mode – If there are sufficient many people on the bridge we are beyond a threshold and synchronization sets in

  (Kuramoto-Synchronizations-Transition, book of Kuramoto in 1984)

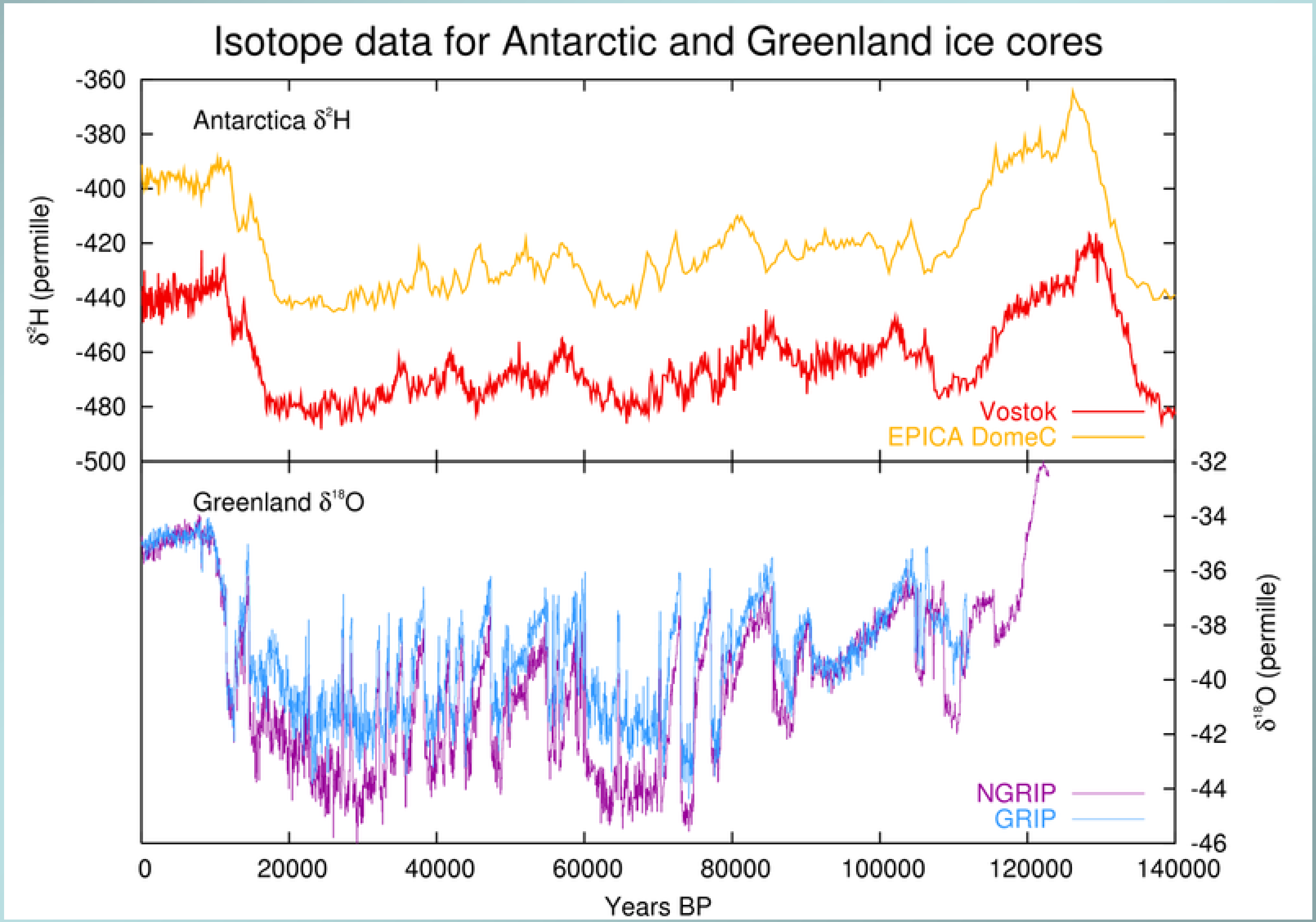
Stabilized afterwards

GERB Schwingungsisolierungen GmbH, Berlin/Essen

Applications & Potentials

- Controlling chaos
- Broad band information transfer
- Ensemble Averaging for Medium Range Weather Forecast – data assimilation
- El Nino – Southern Oscillations (ENSO); Solar Activity – limited predictability
- ENSO – Indian Monsoon synchronization
- Synchronized complex population dynamics (lynx vs. hare dynamics in Canada)
- Dansgaard-Öschger events - SR

Dansgaard-Öschger events



25 events during last glacial period

Limits

Conclusion 5: Restricted to rather low-dimensional systems&only a few aspects of large systems

New challenges from various aspects:

- New era of **spatio-temporal measurement techniques** (satellites, medicine…)
- New era of **communication** (SMS, internet, twitter…)
- Substantially stronger **interrelation** among subsystems

Complex Systems Sciences

Part 2: Complex Networks

(about 2000 - ???)